Permutation Formats

Anyone who ever wants to permute a set will discover the confusion of whether they are moving labels on objects or the objects relative to some natural frame. First, we define a permutation $\sigma \in S^n$ as a bijection $\sigma : \{1, .., n\} \longrightarrow \{1, ..., n\}$.

So called 2-line (Cauchy) notation describes the action of the permutation by

$$\begin{pmatrix} 1 & & n \\ \downarrow & \cdots & \downarrow \\ \sigma(1) & & \sigma(n) \end{pmatrix}.$$

The more brief 1-line notation is just $(\sigma(1), .., \sigma(n))$

follow the arrows to see where elements

go. Then there is cycle notation, which I found necessary to represent permutations on bit sets. In this case, we group sets on which σ acts as a cyclic permutation

$$(i, \sigma(i), \sigma(\sigma(i)), \cdots)(j, \sigma(j), \sigma(\sigma(j)), \cdots)\cdots$$

The conclusion is that 1-line or 2-line notation give us permutations of the <u>values</u> of the integer sets, that is they let us specify that i should go to j; this is convenient to impose a fixed order onto a set. However, cycle notation gives us a convenient platform for moving <u>positions</u> independently of the values they have. Algorithms cannot mix-and-match.